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Horizons and plane waves: A review

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Abstract

We review the attempts to construct black hole/string solutions in asymptotically plane wave spacetimes. First, we demonstrate that geometries admitting a covariantly constant null Killing vector cannot admit event horizons, which implies that pp-waves can't describe black holes. However, relaxing the symmetry requirements allows us to generate solutions which do possess regular event horizons while retaining the requisite asymptotic properties. In particular, we present two solution generating techniques and use them to construct asymptotically plane wave black string/brane geometries.

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1. Introduction

It is an incontrovertible fact that black holes provide an important window into quantum gravity. Knowledge of exact black hole solutions and their properties has greatly enhanced our understanding of gravitational physics and provided us with valuable intuition. Many of the puzzles and paradoxes associated with the unification of general relativity and quantum mechanics have their origins in the fascinating behaviour of black hole event horizons and hence, while we are aware of a rich set of solutions, it is always desirable to extend this list.

Black hole solutions in asymptotically "nice" (*e.g.* maximally symmetric) spacetimes, such as Minkowski, de Sitter, and Anti de Sitter, have been known for decades, and have proved very fruitful in furthering our understanding. A good example is the Schwarzschild-AdS black hole, whose intricacies one may delve into using the well-known AdS/CFT correspondence [1]. Indeed, much recent attention has been devoted to probing behind-the-horizon physics, and especially the singularity, via the CFT [2,3,4,5,6,7]. Unfortunately, all of these efforts have been hampered by the fact that the AdS/CFT is a strong-weak coupling duality; in the regime where supergravity is valid, the gauge theory is strongly coupled, and therefore computationally intractable.

Plane waves, or more generally pp-waves, are another class of spacetimes with rather special properties. Since they admit a covariantly constant null Killing vector, all their curvature invariants vanish identically, rendering these spacetimes exact solutions to classical string theory (with vanishing α' corrections). Nevertheless, they are causally quite nontrivial. For instance, remarkably enough, a large class of these has a one-dimensional null boundary [8,9,10].

It would be therefore very intriguing if these solutions could also admit event horizons, for this would yield exact black hole solutions in classical string theory with rather nontrivial asymptotics. Unfortunately, it turns out that this is not possible, as shown in [11] and reviewed below: pp-waves cannot admit event horizons. This however does not mean that there cannot exist black hole solutions with pp-wave asymptopia. After all, in all of the maximally symmetric spacetimes (flat, dS, and AdS), one also has to break some part of the symmetry in order to obtain black holes in these spacetimes. Indeed, below we will review two distinct methods for obtaining black string solutions which are asymptotically plane wave.

Apart from these solutions representing black holes with interesting asymptopia, they

serve as important steps towards extending the recent BMN correspondence [12]. The BMN correspondence may be viewed as a particular limit of the AdS/CFT correspondence, which on the gravity side generates a maximally supersymmetric plane wave, while on the gauge theory side limits to a large charge sector. This near-BPS regime allows one to do perturbative calculations on both sides of the correspondence, thus allowing greater computational control. It would be therefore very desirable to use this feature to probe the mysteries of quantum gravity, and as the first step, to study black holes in this set-up.

In particular, one may hope that a black hole with the particular plane wave asymptotics considered by [12] would correspond to some tractable modification on the gauge theory side, which we could then use to study such deep issues as singularity resolution, information paradox, etc. So far, such hope is founded mostly on analogy with similar extension of the original AdS/CFT correspondence: Putting a Schwarzschild black hole in AdS spacetime corresponds to thermalizing the gauge theory. Although the BMN regime contains conceptual puzzles of its own, studying black holes in this set-up would be an important endeavour. The requisite solutions have not yet been found; nevertheless, one may be optimistic, as the solutions discussed below already come quite close. In particular, metrically they have the desired asymptotics, though the matter support is different from that arising in the BMN correspondence.

This work does not contain any essentially new results; rather, it summarizes the main points of a series of papers [11], [13], and [14], all motivated by the above goal. We start in the next section by setting notation and establishing the concept of black holes in pp-waves. We then present a simplified proof that pp-waves cannot represent black holes[1] in Section 3. The following two sections discuss black strings in *asymptotically* plane wave spacetimes: extremal solutions with vacuum plane wave asymptotics in Section 4, and more robust, many-parameter family of black string solutions (including Schwarzschild-like one) with maximally symmetric plane wave asymptotics in Section 5. We end with a discussion in Section 6.

2. Definitions

To set the notation and re-emphasize terminology, we will first write down the pp-wave and plane wave metrics, and then discuss what we mean by a black hole with the

[1] We use the terminology black "holes" loosely, to denote all black objects, such as black strings/branes.

corresponding asymptopia.

The pp-wave spacetimes are defined as spacetimes admitting a covariantly constant null Killing field. The metric in d dimensions can be written as

$$ds^2 = -2\, du\, dv - F(u, x^i)\, du^2 + dx^i\, dx^i \ , \tag{2.1}$$

where F is an arbitrary function of u and the transverse coordinates x^i with $i = 1, \cdots, d-2$. The fact that $\left(\frac{\partial}{\partial v}\right)^a$ is a covariantly constant null Killing field ensures that all curvature invariants vanish in these spacetimes. Similarly, in the context of string theory, all the α' corrections vanish.

Plane waves are a special subset of pp-waves, wherein the function F is further restricted to be quadratic in the transverse coordinates, while still maintaining arbitrary u dependence. In particular, plane waves can be written as

$$ds^2 = -2\, du\, dv - f_{ij}(u)\, x^i x^j\, du^2 + dx^i\, dx^i \ . \tag{2.2}$$

These solutions may be thought of as arising from a Penrose limit [15], which essentially zooms in on any null geodesic of any spacetime and reexpands the transverse coordinates. This induces an extra "planar" symmetry along the wavefronts. Further restricting $f_{ij}(u)$ can enhance this symmetry considerably; of greatest import will be the maximally symmetric plane waves, for which $f_{ij}(u) = \mu^2\, \delta_{ij}$, and which have a large isometry group. A particular example of this type of solution is the BFHP plane wave in 10-dimensions [16].

We have discussed the metric, but to specify the full solution, one also needs to specify the matter content. Since the only nonvanishing component of the Ricci tensor is $R_{uu} = \frac{1}{2} \nabla_T^2\, F(u, x^i)$, we see that for vacuum spacetimes, $F(u, x^i)$ must be harmonic in the transverse coordinates. For plane waves, this translates into f_{ij} being traceless, which in turn implies that nontrivial vacuum plane waves cannot be spherically symmetric; and conversely, the maximally symmetric plane waves must have some matter support[2].

Nevertheless, to ascertain the presence of black holes (*i.e.* to find whether or not a given spacetime admits an event horizon), it of course suffices to consider the properties of the metric alone, since that is what determines the causal properties of the solution. Accordingly, in the next section, we will consider general pp-wave metrics without requiring them to be solutions of any particular theory.

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[2] Below, we will use the notation $\mathcal{V}_d$ to denote the vacuum plane waves, and $\mathcal{P}_d$ to denote the maximally symmetric plane waves, in d dimensions.

Before proceeding to prove the absence of black holes in pp-waves, we first need to specify what we mean by black holes. A black hole, by definition, is a region inside event horizon; but an event horizon is well defined only in asymptotically flat spacetime, namely as the boundary of the causal past of the future null infinity (*scri*). In other words, a given asymptotically flat spacetime cannot admit an event horizon if the past of *scri* contains the full spacetime (*i.e.* from every point of the spacetime, there exists a causal curve which reaches *scri*). In more general spacetimes, lacking universally-defined asymptotics, we can try to follow the spirit of this definition by replacing "*scri*" with "arbitrarily far" in spatial directions. While this is generally somewhat murky definition, as we explained in [11], it may be adopted for the pp-waves (2.1) as the following working definition:

Def: A pp-wave spacetime does not admit an event horizon iff from any point in the spacetime, say (u_0, v_0, x_0^i), there exists a future-directed causal curve to some point $(u_1, v_\infty, x_\infty^i)$, where $u_1 > u_0$ is arbitrary, while $v_\infty, x_\infty^i \to \infty$.

The important aspect is that not just v, but also at least one of the transverse coordinates, x^i, gets arbitrarily big along a causal curve[3]. We will in fact use a stronger version of this criterion; namely, we will require $u_1 = u_0 + \varepsilon$, for arbitrarily small $\varepsilon > 0$. This allows us to use the criterion in greater generality, in particular even in the cases where the spacetime terminates at some finite u, *i.e.*, $F(u, x^i) \to \infty$ as $u \to u_\infty < \infty$.

3. No horizons in pp-waves

We will now use the above definition to demonstrate why pp-waves cannot admit horizons, and therefore cannot be black holes. This was motivated for plane waves and proved for general pp-waves in [11]; here we will present a simplified version of the proof, which applies for all spacetimes (2.1) with $F(u, x^i) \geq 0$, and refer the reader to [11] for the more general proof.

The lack of horizons in plane waves is very simple to understand intuitively: plane waves have a planar symmetry which precludes any special position at which the horizon could be[4]. Alternately, one can argue that any plane wave is a Penrose limit of some

[3] Since the fact that $\left(\frac{\partial}{\partial v}\right)^a$ is null and Killing implies that it describes orbits of null geodesics moving in the v direction, the criterion that from any point in such a spacetime there is a causal curve attaining arbitrarily large v is trivially satisfied. Furthermore, the crucial fact that the coordinate chart used in (2.1) covers the full spacetime makes this definition meaningful.

[4] This of course doesn't mean that there can't be observer-dependent horizons, such as in

spacetime, but event horizons cannot be retained under a Penrose limit, as the latter loses the global information about the spacetime. Hence, we will present a simplified version of the no-horizon proof for the more general pp-wave spacetimes; the fact that plane waves can't have horizons then follows immediately as a corollary.

As indicated above, in order to demonstrate the absence of horizons, it suffices to show that from *any* point $p_0 = (u_0, v_0, x_0^i)$ of the spacetime, there exists a future-directed, causal curve γ (*i.e.* one along which $\dot{u} > 0$ and $-2\,\dot{u}\,\dot{v} - F(u,x^i)\,\dot{u}^2 + \dot{x}^i\,\dot{x}^i \leq 0$) which reaches arbitrarily large values of r and v in arbitrarily small Δu. One can construct such a curve γ explicitly, as demonstrated in [11]; however, there is a more powerful and elegant technique which proceeds by introducing a fiducial metric, with smaller "light cones" than those of the physical spacetime we are interested in. Any curve which is causal in the fiducial metric will then also have to be causal in the physical one. The advantage of this observation is that by selecting appropriate fiducial metric, the construction of causal curves with the requisite properties is rendered much easier.

If $F(u,x^i)$ is non-negative everywhere, such a convenient fiducial metric is simply the flat spacetime, which corresponds to the pp-wave metric (2.1) with $F(u,x^i) \equiv 0$. This translates into finding a curve $\gamma(\lambda)$ such that

$$\begin{aligned} &\gamma(0) = (u_0, v_0, x_0^i) \\ &\gamma(1) = (u_0 + \varepsilon, v_\infty, x_\infty^i) \\ \text{and} \qquad &-2\,\dot{u}\,\dot{v} + \dot{x}^i\,\dot{x}^i \leq 0 \end{aligned} \tag{3.1}$$

In flat spacetime, this is easily achieved by considering a "straight line" between the points $p_0 \equiv \gamma(0)$ and $p_\infty \equiv \gamma(1)$, and the causal relation simply integrates to

$$-2\,\Delta u\,\Delta v + \Delta x^i\,\Delta x^i \leq 0 \tag{3.2}$$

The desired causal curve can then be obtained by the following series of steps:

- Pick any point $p_0 = (u_0, v_0, x_0^i)$ in the spacetime.
- Choose arbitrarily large x_∞^i (or equivalently, arbitrarily large Δx^i).
- Choose $\Delta u = \varepsilon$ arbitrarily small (but positive, so that γ is future-directed).
- Since (3.2) implies that γ will be causal if $\Delta v \geq \frac{\Delta x^i\,\Delta x^i}{2\,\varepsilon}$, choose arbitrarily large v_∞ such that

$$v_\infty \geq \frac{\Delta x^i\,\Delta x^i}{2\,\varepsilon} + v_0\ . \tag{3.3}$$

Rindler space or de Sitter; however, since Rindler horizons exist in any causally well-behaved spacetime, here we shall be concerned only with the black hole "event" horizons.

This construction then automatically gives the requisite causal curve in the physical spacetime (2.1) as well, thereby proving that pp-waves cannot admit event horizons. As mentioned earlier, [11] generalizes this proof to arbitrary $F(u, x^i)$.

The no-go theorem for horizons in pp-wave metrics is rather disappointing from the standpoint of finding black holes in string theory described by exact classical solutions which receive no α' corrections. However, this does not necessarily imply that there are no such solutions, since the pp-waves are not the most general spacetimes with vanishing curvature invariants; cf. *e.g.* [17,18,19]. It would be interesting to repeat our analysis for these more general spacetimes.

4. Horizons in spacetimes with null isometry

Having shown that spacetimes with a covariantly constant null Killing field (pp-waves) do not admit event horizons, we can now ask whether relaxing the covariantly constant requirement enables us to construct spacetimes with horizons which nevertheless have a globally null Killing vector. Naively, one might expect that even null isometry is too restrictive to allow for horizons; but this is clearly not the case, as there are well-known examples of extremal asymptotically flat black branes with null isometry.

Although these solutions are asymptotically flat, it is not difficult to write down asymptotically vacuum plane wave black branes, as discussed in [13] and as we briefly review below. The "trick" is to use a solution generating technique, originally developed by [20,21], called the Garfinkle-Vachaspati (GV) method. Given a solution to Einstein's equations (in general, with some appropriate matter content), which admits a hypersurface-orthogonal null Killing field compatible with the matter content, one can deform the solution to a new one with the same matter fields. In particular, the curvature invariants of the deformed solution are identical to the parent solution. The idea is that given a solution with an appropriate set of symmetries, one can essentially "linearize" Einstein's equations, which will then allow one to superpose solutions.

A trivial example of using the GV technique is to apply it to flat space to obtain a vacuum plane wave, or more generally, a vacuum pp-wave. In particular, starting with the metric (2.2) (with $f = 0$ for flat space), the GV solution generating technique essentially amounts to adding a term $F(u, x^i)\, du^2$, where F is harmonic, to the metric. This by definition corresponds to a pp-wave. To use the GV construction for the problem at hand, the strategy is to start with a solution with a regular horizon and add a term which

preserves the integrity of the horizon, but changes the asymptotics in the required fashion. As mentioned above, the (asymptotically flat) extremal black branes in ten (IIA/IIB) or eleven dimensional supergravity satisfy the necessary criteria for the GV construction to be applicable; but not all have a regular horizon. Specifically, only the D3, M2, and M5 branes have a regular horizon, and of these, only the M2-brane solution is capable of producing a spacetime wherein the regular event horizon cloaks a singularity; so we will restrict our attention to this case (the other cases were briefly discussed in [13]).

To deform the M2-brane solution in eleven dimensional supergravity so that the asymptotic behaviour is $\mathcal{V}_{10} \times \mathbf{R}$, whilst retaining the nature of the near-horizon geometry and the singularity intact, we proceed as follows. We start with the M2-brane solution to 11-dimensional supergravity, given by

$$
\begin{aligned}
ds^2 &= H(r)^{-\frac{2}{3}} \left(-2\,du\,dv + dx^2\right) + H(r)^{\frac{1}{3}} \left(dr^2 + r^2\,d\Omega_7^2\right) \\
F_4 &= \left(\frac{dH(r)^{-1}}{dr}\right) du \wedge dv \wedge dx \wedge dr.
\end{aligned}
\tag{4.1}
$$

where we have combined time with one of the spatial directions along the brane to write the metric in a more convenient form; to wit, both $\left(\frac{\partial}{\partial v}\right)^a$ and $\left(\frac{\partial}{\partial u}\right)^a$ are null Killing vectors and we can also verify that they are hypersurface-orthogonal. $H(r) = 1 + \frac{Q^6}{r^6}$ is a harmonic function in the transverse eight-dimensional space. The horizon is at $r = 0$ in these coordinates. To see the location of the singularity, it is best to define a new coordinate $\zeta = r^2$. The singularity is located at the zero of $H(\zeta)$, *i.e.*, $\zeta = -Q^2$.

We can now apply the GV construction to write a new ("deformed") metric as

$$
ds^2 = H(r)^{-\frac{2}{3}} \left(-2\,du\,dv + dx^2 - \Psi(u,x,r,\Omega_7)\,du^2\right) + H(r)^{\frac{1}{3}} \left(dr^2 + r^2\,d\Omega_7^2\right) \tag{4.2}
$$

where the new term Ψ must be harmonic, $\nabla^2\,\Psi = 0$. Writing $\Psi(u,x,r,\Omega_7) = \xi_{kL}(u)\,e^{ikx}\,\psi_{k\ell}(r)\,Y_L(\Omega_7)$ with arbitrary functions $\xi_{kL}(u)$, and $Y_L(\Omega_7)$ denoting the spherical harmonics with L being a label for the set of angular momenta on the seven sphere with principal angular momentum ℓ, we find the radial equation

$$
\frac{d^2\psi_{k\ell}(r)}{dr^2} + \frac{7}{r}\,\frac{d\psi_{k\ell}(r)}{dr} - \left(\frac{\ell(\ell+6)}{r^2} + k^2\,H(r)\right)\psi_{k\ell}(r) = 0 \tag{4.3}
$$

For the case of $k = 0$, *i.e.*, requiring $\left(\frac{\partial}{\partial x}\right)^a$ to be a Killing vector in the deformed geometry, the problem reduces to solving the Laplace equation in eight dimensional flat space. So we can pick the $\ell = 2$ mode on the seven sphere to obtain a solution which is asymptotically plane wave. For example, parameterizing the seven sphere by the coordinates such

that θ corresponds to the azimuthal angle, we can choose $\Psi(u,x,r,\Omega_7) = r^2\,\left(8\cos^2\theta - 1\right)$. Then

$$\begin{aligned} ds^2 &= H(r)^{-\frac{2}{3}}\,\left[-2\,du\,dv + dx^2 - r^2\left(8\,\cos^2\theta - 1\right)\,du^2\right] \\ &\quad + H(r)^{\frac{1}{3}}\,\left[dr^2 + r^2\left(d\theta^2 + \sin^2\theta\,d\Omega_6^2\right)\right] \\ F_4 &= \left(\frac{dH(r)^{-1}}{dr}\right)\,du\wedge dv\wedge dx\wedge dr, \end{aligned} \tag{4.4}$$

is a solution to eleven dimensional supergravity, with the same curvature invariants as the M2-brane solution. In particular, the solution still has a regular horizon[5] at $r = 0$. Furthermore, its asymptotic behaviour is that of a ten dimensional vacuum plane wave times an extra real line parameterized by x.

In passing, we note that when $k \neq 0$, the nature of the asymptotics changes quite dramatically. The asymptotic behaviour of the equation (4.3) can be analyzed to show that the solutions are Bessel functions, which are incompatible with the necessary r^2 behaviour. So by looking for solutions wherein we have some momentum along the brane world-volume directions we do not obtain a solution that looks like a plane-wave. In order to obtain a plane wave solution, our only choice is then to use the solution presented in (4.4), wherein we have an additional isometry corresponding to translations along $\left(\frac{\partial}{\partial x}\right)^a$. We note that extensions of the GV technique were also considered in [25], [26], and [27].

Although the above solution (4.4), where we could compactify along the x direction, is interesting in its own right, in the bigger scheme of attempting to extend the BMN correspondence to BFHP black hole spacetimes, it falls short in two important aspects: it corresponds to an extremal, rather than a neutral, black hole; and the asymptotic region corresponds to a vacuum, rather than the maximally symmetric BFHP, plane wave. In the next section we will rectify the first shortcoming completely and the second one at least partially; but before presenting that construction, we will make a few more remarks.

As a first step to correcting the above-mentioned deficiencies, one can try to look for a neutral black hole solution in asymptotically vacuum plane wave spacetime. One would expect that there be such a solution, since after all, one might achieve it physically by colliding two oppositely charged extremal black strings discussed above. Indeed, assuming the same symmetries as for the "seed" asymptotically flat extremal black string (namely globally null Killing vector and transverse spherical symmetry), all vacuum solutions can

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[5] This is to be contrasted with added pp-wave like terms $\Psi(u,r) \sim \frac{1}{r}$, or having plane waves along the longitudinal directions of the brane, wherein one does encounter singularities at the horizon *cf.*, [22], [23], [24]

be found analytically. This was done in [13], where the new vacuum solutions were written down explicitly and their properties analysed. Interestingly, it turns out that (independently of their asymptotic behaviour) none of these solutions can admit horizons; they are in fact nakedly singular. This exemplifies the remarkable point that restricting ourselves to vacuum Einstein's equations severely restricts the nature of the possible solutions, and in particular eliminates the causally nontrivial ones.

At the first sight, this result may seem paradoxical in light of the naive argument of constructing vacuum black strings by colliding oppositely-charged ones, but actually there is no contradiction: the collision process would spoil the null symmetry. In fact, this might be expected already from the explicit construction outlined above: only extremal black branes admit a null symmetry, so we can't start with a non-extremal solution instead.

The second shortcoming, that the asymptotics correspond to vacuum, rather than the maximally symmetric plane wave (in the transverse directions), is in fact a necessary consequence of using the GV construction to generate the solution from asymptotically flat spacetime: the matter content is unchanged, so vacuum asymptotics can only lead to vacuum asymptotics. In the next section, this will be contrasted by a different solution generating technique, which in fact generates the maximally symmetric (nonvacuum) plane wave starting from the flat space.

5. Plane wave black strings

In the previous section we have seen that there exist asymptotically vacuum plane wave solutions with horizons and null isometry; however, the black branes were extremal and the asymptopia non-maximally symmetric. To do better, we should further relax the required symmetries of the solution; namely, we now drop the globally null isometry. As is often the case, the price we pay for breaking symmetries is manifested in the increased difficulty of finding the solutions. Solving the ansatz by brute-force is forbiddingly messy; but fortunately, one can once again resort to an elegant trick of using a particular solution generating technique, which we call the "Null Melvin Twist" [14].

Our method is based on the observation of [28] that certain class of plane wave geometries can be generated by applying a sequence of manipulations to Minkowski space. By applying the same sequence of manipulations starting from a black string solution, we are able to generate a large class of black string deformations of plane wave geometries with a regular horizon. These solutions are characterized by the mass density of the black

string and the scale of the plane wave geometry in the rest frame of the black string. For dimensions greater than six, the deformation due to the presence of the black string decays at large distances in the direction transverse to the string. Hence in these cases, our solutions describe geometries with (radially) plane wave asymptotics. However, the solutions described will be plane-wave geometries which are supported by 3-form flux rather than 5-form flux as is desirable for the BMN correspondence.

The specific set of steps is quite simple, and requires only a starting 10 dimensional supergravity solution (in NS-NS sector) with a constant-norm isometry $\left(\frac{\partial}{\partial y}\right)^a$. One then performs the following sequence of steps:

- Boost the space-time along y by γ
- T-dualize along the y coordinate
- Twist by making a change of coordinates[6] $\sigma \to \sigma + 2\,\alpha\,dy$
- T-dualize back along y
- Boost back along y by $-\gamma$
- Finally, perform a double scaling limit, wherein the boost γ is scaled to infinity and the twist α to zero keeping $\beta \equiv \frac{1}{2}\,\alpha\,e^{\gamma} =$ fixed.

As mentioned above, starting with the 10 dimensional Minkowski spacetime, $ds^2_{str} = -dt^2 + dy^2 + dr^2 + r^2\,d\Omega_7^2$ with $\Phi = 0,\ B = 0$, we obtain the plane wave

$$\begin{aligned} ds^2_{str} &= -(1+\beta^2\,r^2)\,dt^2 - 2\,\beta^2\,r^2\,dt\,dr + (1-\beta^2\,r^2)\,dy^2 + dr^2 + r^2\,d\Omega_7^2 \\ \Phi &= 0, \qquad B = \frac{\beta\,r^2}{2}\,(dt+dy)\wedge\sigma \end{aligned} \tag{5.1}$$

where the usual form is obtained by letting $u = t + y$ and $2v = t - y$.

Similarly, starting with the asymptotically flat Schwarzschild black string solution,

$$ds^2_{str} = -f(r)\,dt^2 + dy^2 + \frac{1}{f(r)}\,dr^2 + r^2\,d\Omega_7^2 \tag{5.2}$$

one obtains

$$\begin{aligned} ds^2_{str} &= -\frac{f(r)\,\left(1+\beta^2\,r^2\right)}{k(r)}\,dt^2 - \frac{2\,\beta^2\,r^2\,f(r)}{k(r)}\,dt\,dy + \left(1 - \frac{\beta^2\,r^2}{k(r)}\right)\,dy^2 \\ &\quad + \frac{dr^2}{f(r)} + r^2\,d\Omega_7^2 - \frac{\beta^2\,r^4\,(1-f(r))}{4\,k(r)}\,\sigma^2 \\ e^{\Phi} &= \frac{1}{\sqrt{k(r)}}\,, \qquad B = \frac{\beta r^2}{2k(r)}\,(f(r)\,dt + dy)\wedge\sigma \end{aligned} \tag{5.3}$$

[6] Here we write the metric on the $\mathbf{S}^7$ in terms of polar coordinates $d\Omega_7^2 = d\chi^2 + \frac{1}{4}\,\cos^2\chi\,d\Omega_a^2 + \frac{1}{4}\,\sin^2\chi\,d\Omega_b^2$ with $d\Omega_i^2 \equiv d\theta_i^2 + d\psi_i^2 + d\phi_i^2 + 2\cos\theta_i\,d\psi_i\,d\phi_i,\ i = a, b$, and introduce a symmetric 1-form $\sigma \equiv \cos^2\chi\,(\cos\theta_1\,d\psi_1 + d\phi_1) + \sin^2\chi\,(\cos\theta_2\,d\psi_2 + d\phi_2)$.

where $f(r) \equiv 1 - \frac{M}{r^6}$ and $k(r) \equiv 1 + \frac{\beta^2 M}{r^4}$.

The solution (5.3) is very simple. By inspection, if we set M to zero the solution reduces to the maximally symmetric plane wave $\mathcal{P}_{10}$. On the other hand, setting $\beta = 0$ will reduce the solution to the black string solution (5.2). For all finite values of M, there is a curvature singularity at $r = 0$, which can be easily checked to be spacelike. More importantly, there is a regular horizon at

$$r_H = M^{1/6} \tag{5.4}$$

which persists for finite values of β. One can therefore interpret (5.3) as the black string deformation of $\mathcal{P}_{10}$. Furthermore, since both $f(r)$ and $k(r)$ asymptote to 1 as r is taken to be large, the effect of M decays at large r. Unlike the six dimensional solution described in [29] which deformed the geometry by a finite amount at large r, (5.3) is a black string solution which genuinely asymptotes to $\mathcal{P}_{10}$ (in the transverse directions).

A rather remarkable property of this solution concerns its thermodynamics. As discussed in [14], the area of the horizon in Einstein frame metric is given by

$$\mathcal{A}_H = L\, M^{7/6}\, \Omega_7 \ , \tag{5.5}$$

while a natural definition[7] of temperature would evaluate to

$$T_H = \frac{3}{2\pi} M^{-1/6} \ . \tag{5.6}$$

The first law of thermodynamics would then naturally allow us to interpret the parameter M as the mass density of the black string. The remarkable fact about these quantities is that they are independent of the parameter β, *i.e.* the plane wave black string thermodynamics is the same as the usual asymptotically flat black string thermodynamics.

As discussed in [14], the solution (5.3) can be generalized to include rotations, charge, or more general twists, giving rise to a 13-parameter family of solutions, all with regular horizons and maximally symmetric plane wave asymptotics. Furthermore, one can also repeat the construction starting with black Dp-branes, which generates plane wave $\mathcal{P}_{11-p}$ asymptotics transverse to the brane.

Although these are all intriguing generalizations with diverse and interesting properties, for the purposes of extending the BMN story, it would be far more useful to generate

[7] As discussed in [14], there is a normalization ambiguity.

solutions with asymptotics which are not only metrically $\mathcal{P}_{10}$, but also have the requisite matter content, namely the RR 5-form field strength. In our construction, we explicitly exploited the usual duality between off-diagonal metric components and magnetic fields, which is why we obtained solutions supported by NS-NS flux. (In IIB it is trivial to S-dualise to generate solutions supported by a combination of NS-NS and RR 3-form fluxes). However, since there is no way to geometrically engineer RR 5-form fluxes, we will have to work harder to generate solutions supported by the same.

Nevertheless, it is tempting to conjecture that the final solutions will have similar thermodynamic properties as the solutions described above. This observation is simply based on the fact that the deformation of asymptotics from flat space to plane wave form seems to have no apparent effect on the thermal properties.

6. Discussion

We have summarized a particular development of ideas, to date still incomplete, to construct asymptotically BMN black hole spacetimes. Having shown that pp-wave (or more specifically, plane wave) metrics cannot represent black holes, we have reviewed the construction of asymptotically plane wave black holes. The first solution-generating method (the GV construction) we used relied heavily on null isometry and preserved the matter content of the seed solution. As such, it only allowed us to construct extremal black strings with vacuum plane wave asymptopia. While interesting in its own right, a far more fruitful technique proved to be the Null Melvin Twist, which allowed us to generate a wide class of black strings with the metric asymptoting to the desired maximally symmetric plane wave. However, even in the latter construction, we only managed to generate solutions that were supported by 3-form fluxes (in ten dimensional supergravity).

There are many open questions that remain to be discussed in this general story. One obvious extension is to obtain solutions that have 5-form flux support. Another would be to find explicit black hole solutions rather than the black string solutions mentioned above. These appear to be sufficiently complicated to be tackled by brute-force solution of Einstein equations and we are not aware of any appropriate solution generating technique. If such a solution generating technique were to be uncovered, its utility in constructing general warped flux compactifications would be enormous.

At the same time there are many interesting avenues that are amenable to further study taking off from the solutions presented here. For instance, it is very intriguing

that the plane wave deformation of the black string geometries appears to be "irrelevant" and it would be worthwhile to investigate the generality of this result. One should also study the causal properties of black strings in plane wave geometries to figure out whether there are any remnants of the exotic features seen in the causal structures of plane waves. Furthermore, these solutions provide an excellent setting to understand the notions of ADM mass and other conserved charges in backgrounds which are asymptotically plane wave.

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